# Static Single Assignment Form 

CMPT 379: Compilers<br>Instructor: Anoop Sarkar<br>anoopsarkar.github.io/compilers-class

## SSA Form

- Conversion from a Control Flow Graph (created from 3-address code) into SSA Form is not trivial
- SSA creation algorithms:
- Original algorithm by Cytron et al. 1986
- Lengauer-Tarjan algorithm (see the Tiger book by Andrew W. Appel for more details)
- Harel algorithm


## Conversion to SSA Form

- Simple idea: add a $\phi$ function for every variable at a join point
- A join point is any node in the control-flow graph with more than one predecessor
- But: this is wasteful and unnecessary.


## Conversion to SSA Form



## Conversion to SSA Form



Conversion to SSA Form (with minimal $\phi$ functions)

## Dominance Relation

- X dominates Y if every path from the start node to Y goes through X
- $D(X)$ is the set of nodes that $X$ dominates
- X strictly dominates Y if X dominates Y and $\mathrm{X} \neq \mathrm{Y}$


## Dominance Relation



## Dominance Relation



## Dominance Property of SSA

- Essential property of SSA form is the definition of a variable must dominate use of the variable:
- If variable $a$ is used in a $\phi$ function in block $X$, then definition of $a$ dominates every predecessor of $X$
- If $a$ is used in a non- $\phi$ statement in block $X$, then the definition of $a$ dominates X .


## Dominance Relation



## Dominance Relation



## Dominance Frontier

- X strictly dominates Y if X dominates Y and $\mathrm{X} \neq \mathrm{Y}$
- Dominance Frontier (DF) of node X is the set of all nodes Y such that:
- $X$ dominates a predecessor of Y , and
- X does not strictly dominate Y


## Dominance Frontier



## Dominance Frontier

- Algorithm to compute $\operatorname{DF}(\mathrm{X})$ :
- Local(X) := set of successors of $X$ that $X$ does not immediately dominate
- $\mathrm{Up}(\mathrm{X}):=$ if X dominates $\mathrm{K}, \mathrm{Up}(\mathrm{X})$ is the set of nodes in $\mathrm{DF}(\mathrm{K})$ that are not dominated by X .
- DF(X) := Union of Local $(\mathrm{X})$ and ( Union of $\mathrm{Up}(\mathrm{K})$ for all K that are children of $X$ )


## ComputeDF(5)

## Dominance Frontier

- ComputeDF(X): $\{6,7\}$
$S:=\{ \} / /$ empty set
For each node $Y$ in Successor(X):
If $X$ does not strictly dominate $Y$ :

$\mathrm{S}:=\mathrm{S} \cup\{\mathrm{Y}\} / /$ this is Local $(\mathrm{X})$, U means union
For each child $K$ of $X$ in $D(X)$ : // $X$ dominates $K$,
For each element $Y$ in ComputeDF(K):
If $X$ does not dominate $Y$,

$$
S:=S \cup\{Y\} / / \text { this is } U p(X)
$$

DF(X) = S; return S


## Dominance Frontier

- Dominance Frontier Criterion
- If node X contains definition of some variable $a$, then any node Y that uses $a$ in the set $\operatorname{DF}(X)$ needs a $\phi$ function for $a$.
- Iterated Dominance Frontier
- Since a $\phi$ function is itself a definition of a new variable, we must iterate the DF criterion until no nodes in the CFG need a $\phi$ function.


## Placing $\phi$ Functions

$$
\mathrm{DF}(3)=\{7\}
$$



## Placing $\phi$ Functions



## Placing $\phi$ Functions



$$
D F(5)=\{6\}
$$

## Placing $\phi$ Functions

$$
\mathrm{DF}(6)=\{7\}
$$



Rename Variables
$D F(6)=\{7\}$


## Summary

- Compute the dominance frontier for each node in the flowgraph
- For each node $X$ place a $\phi$ function in each node that is in the dominance frontier for $X$
- Iterate the dominance frontier algorithm above for each new variable assignment in each $\phi$ function added in the previous step
- The end result: 3-address code converted into Static Single Assignment (SSA) form

